Soluciones de cálculo de límites de sucesiones II

4 $\lim(\sqrt{n^2-2}-\sqrt{n^2+n})$

$$\lim(\sqrt{n^2-2}-\sqrt{n^2+n}) = \infty - \infty$$

$$\lim\frac{\left[(\sqrt{n^2-2}-\sqrt{n^2+n})(\sqrt{n^2-2}+\sqrt{n^2+n})\right]}{(\sqrt{n^2-2}+\sqrt{n^2+n})} =$$

$$= \lim\frac{n^2-2-n^2-n}{(\sqrt{n^2-2}+\sqrt{n^2+n})} = \lim_{x\to\infty}\frac{-2-n}{(\sqrt{n^2-2}+\sqrt{n^2+n})} =$$

$$= \lim\frac{\frac{-2}{n}-\frac{n}{n}}{\sqrt{\frac{n^2}{n^2}-\frac{2}{n^2}}+\sqrt{\frac{n^2}{n^2}+\frac{n}{n^2}}} = \frac{-1}{1+1} = \frac{-1}{2}$$

5 $\lim\left(\sqrt{n^2+3n}-\sqrt{n^2+n}\right)$

$$\lim\left(\sqrt{n^2+3n}-\sqrt{n^2+n}\right) = \infty - \infty$$

$$= \lim\frac{\left(\sqrt{n^2+3n}-\sqrt{n^2+n}\right)\cdot\left(\sqrt{n^2+3n}+\sqrt{n^2+n}\right)}{\left(\sqrt{n^2+3n}+\sqrt{n^2+n}\right)} =$$

$$= \lim\frac{n^2+3n-n^2-n}{\left(\sqrt{n^2+3n}+\sqrt{n^2+n}\right)} = \lim\frac{2n}{\left(\sqrt{n^2+3n}+\sqrt{n^2+n}\right)} = \frac{\infty}{\infty}$$

UNPROFESOR

$$= \lim \frac{\frac{2n}{n}}{\sqrt{\frac{n^2}{n^2} + \frac{3}{n^2}} + \sqrt{\frac{n^2}{n^2} + \frac{n}{n^2}}} = \lim \frac{2}{\sqrt{1 + \frac{3}{n^2}} + \sqrt{1 + \frac{1}{n}}} = \frac{2}{2} = 1$$

Ejercicio 10 resuelto

Hallar los límites:

Soluciones:

1 $\lim (n+7) \cdot \sqrt{\frac{1}{4n^2+3}}$

$$\lim (n+7) \cdot \sqrt{\frac{1}{4n^2+3}} = \infty \cdot 0$$

Introducimos el 1er factor en la raíz.

$$\lim \sqrt{\frac{(n+7)^2}{4n^2+3}} = \frac{\infty}{\infty}$$

$$\sqrt{\lim \frac{n^2+14n+49}{4n^2+3}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

2 $\lim \frac{\frac{3}{\sqrt{4n^2+5}}}{\frac{1}{n-1}}$

$$\lim \frac{\frac{3}{\sqrt{4n^2+5}}}{\frac{1}{n-1}} = \frac{0}{0}$$

UNPROFESOR

Se transforma a $\frac{\infty}{\infty}$

$$\lim \frac{\frac{3}{\sqrt{4n^2+5}}}{\frac{1}{n-1}} = \lim \frac{3n-3}{\sqrt{4n^2+5}} = \lim \frac{\frac{3n}{n}-\frac{3}{n}}{\sqrt{\frac{4n^2}{n^2}+\frac{5}{n^2}}} = \lim \frac{3-\frac{3}{n}}{\sqrt{4+\frac{5}{n^2}}} = \frac{3}{2}$$

$$\lim\left(3n \cdot \frac{2n}{n^2-7n-5}\right) = \infty \cdot 0$$

$$\lim \frac{6n^2}{n^2-7n-5} = 6$$

3 $$\lim\left(\sqrt{18n^2+1} \cdot \frac{1}{\sqrt{32n^2-3}}\right)$$

$$\lim\left(\sqrt{18n^2+1} \cdot \frac{1}{\sqrt{32n^2-3}}\right) = \infty \cdot 0$$

$$\lim \sqrt{\frac{18n^2+1}{32n^2-3}} = \sqrt{\lim \frac{18n^2+1}{32n^2-3}} = \sqrt{\frac{18}{32}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

4 $$\lim \frac{\frac{3}{n^2+2}}{\sqrt{\frac{7}{n^2-1}}}$$

$$\lim \frac{\frac{3}{n^2+2}}{\sqrt{\frac{7}{n^2-1}}} = \frac{0}{0}$$

Ejercicio 11 resuelto

Calcula los siguienteslímites:

Soluciones:

1 $\lim\left(\frac{2n^2}{3n+1}\right)^{\frac{3n^2+2}{5n-3}}$

$$\lim\left(\frac{2n^2}{3n+1}\right)^{\frac{3n^2+2}{5n-3}} = \infty^{\infty} = \infty$$

2 $\lim\left(\frac{2n^2}{3n+1}\right)^{\frac{-3n^2+2}{5n-3}}$

$$\lim\left(\frac{2n^2}{3n+1}\right)^{\frac{-3n^2+2}{5n-3}} = \infty^{-\infty} = \frac{1}{\infty^{\infty}} = \frac{1}{\infty} = 0$$

3 $\lim\left(\frac{2n^2}{3n+1}\right)^{\frac{-3n^2+2}{5n^2-3}}$

$$\lim\left(\frac{2n^2}{3n+1}\right)^{\frac{-3n^2+2}{5n^2-3}} = \infty^{-\frac{3}{5}} = \frac{1}{\sqrt[5]{\infty^3}} = \frac{1}{\infty} = 0$$

4 $\lim\left(\frac{2n^2}{3n^3+1}\right)^{\frac{3n^2+2}{5n-3}}$

$$\lim\left(\frac{2n^2}{3n^3+1}\right)^{\frac{3n^2+2}{5n-3}} = 0^{\infty} = 0$$

5 $$\lim\left(\frac{2n^2}{3n^3+1}\right)^{\frac{-3n^2+2}{5n-3}}$$

$$\lim\left(\frac{2n^2}{3n^3+1}\right)^{\frac{-3n^2+2}{5n-3}} = 0^{-\infty} = \frac{1}{0^{\infty}} = \frac{1}{0} = \infty$$

6 $$\lim_{x\to\infty}\left(\frac{2n^2}{3n^2+1}\right)^{\frac{-3n+2}{5n^2-3}}$$

$$\lim\left(\frac{2n^2}{3n^2+1}\right)^{\frac{-3n+2}{5n^2-3}} = \left(\frac{2}{3}\right)^{0} = 1$$

7 $$\lim\left(\frac{2n^2}{3n^2+1}\right)^{\frac{-3n^2+2}{5n-3}}$$

$$\lim\left(\frac{2n^2}{3n^2+1}\right)^{\frac{-3n^2+2}{5n-3}} = \left(\frac{2}{3}\right)^{-\infty} = \left(\frac{3}{2}\right)^{\infty} = \infty$$

8 $$\lim\left(\frac{2n^2}{3n^2+1}\right)^{\frac{3n^2+2}{5n-3}}$$

$$\lim\left(\frac{2n^2}{3n^2+1}\right)^{\frac{3n^2+2}{5n-3}} = \left(\frac{2}{3}\right)^{\infty} = 0$$

$$\lim\frac{3\sqrt{n^2-1}}{\sqrt{7}\left(n^2+2\right)} = 0$$

Ejercicio 12 resuelto

Calcula los siguienteslímites:

Soluciones:

1 $\lim\left(1+\frac{1}{n+2}\right)^{n-1}$

$$= \lim\left(1+\frac{1}{n+2}\right)^{n+2-2-1} =$$

$$= \lim\left(1+\frac{1}{n+2}\right)^{n+2-3} =$$

$$= \frac{\lim\left(1+\frac{1}{n+2}\right)^{n+2}}{\lim\left(1+\frac{1}{n+2}\right)^{3}} = \frac{e}{1} = e$$

2 $\lim\left(1-\frac{2}{3n}\right)^{n}$

$$\lim\left(1-\frac{2}{3n}\right)^{n}=1^{\infty}$$

$$=\lim\left(1+\frac{-2}{3n}\right)^{n}=\lim\left(1+\frac{1}{\frac{3n}{-2}}\right)^{n}=$$

$$=\left[\lim\left(1+\frac{1}{\frac{3n}{-2}}\right)^{\frac{3n}{-2}}\right]^{\lim\frac{-2}{3n}\cdot n}=e^{\frac{-2}{3}}=\frac{1}{\sqrt[3]{e^{2}}}$$

3 $\lim\left(\frac{2n+1}{2n+4}\right)^{\frac{n^2}{n+1}}$

$$\lim\left(\frac{2n+1}{2n+4}\right)^{\frac{n^2}{n+1}}=1^{\infty}$$

$$=\lim\left(1+\frac{-3}{2n+4}\right)^{\frac{n^2}{n+1}}=$$

$$=\lim\left(1+\frac{1}{\frac{2n+4}{-3}}\right)^{\frac{n^2}{n+1}}=$$

$$= \left[\lim \left(1 + \frac{1}{\frac{2n+4}{-3}} \right)^{\frac{2n+4}{-3}} \right]^{\lim \frac{-3}{2n+4} \cdot \frac{n^2}{n+1}} = e^{\frac{-3}{2}} = \frac{1}{\sqrt{e^3}}$$